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$=3(R-h)^2(2R+h)/6R^3=m/n$. Therefore, $(h/R)^3-3(h/R)+2=2m/n$; or, $(h/R)^3-3(h/R)+2(n-m)/n=0$. Applying the proper trigonometric formula, $h/R=2\sqrt{(p/3)}\sin\frac{1}{3}\theta$, where $p=3$, $\theta=\sin^{-1}3q/2p\sqrt{(3/p)}=\sin^{-1}(n-m)/n$, $q=2(n-m)/n$. Hence, $h/R=2\sin\frac{1}{3}[\sin^{-1}(n-m)/n]$.

194. Proposed by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

Glass paper weights, having the form of a regular tetrahedron, are to be packed for shipment, each in a paper box. Wanted to know the size and shape of the smallest box for the purpose. How much empty space in each box?

Solution by the PROPOSER.

Shape of box=cube.

Edge of box$=s\sqrt{\frac{1}{2}}$, where s=edge of tetrahedron.

Empty space$=\frac{1}{3}s^3\sqrt{\frac{1}{2}}$.

Occupied space$=\frac{1}{6}s^3\sqrt{\frac{1}{2}}$=volume of tetrahedron.

Total space$=\frac{1}{2}s^3\sqrt{\frac{1}{2}}$=volume of box.

Of the twelve diagonals in the six faces of the box, the six edges of the tetrahedron coincide with one in each face.

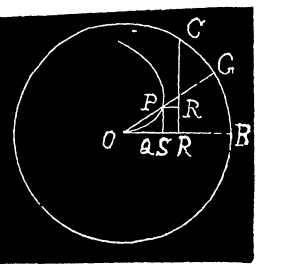
CALCULUS.

160. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A dog at the vertex of a right conical hill pursues a fox at the foot of the hill. How far will the dog run to catch the fox, if the dog runs directly towards the fox at all times, and the fox is continually running around the hill at its foot, the velocity of the dog being 6 feet per second, the velocity of the fox being 5 feet per second, the hill being 100 feet high and 200 feet in diameter at the base?

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let the origin be the vertex of the cone, O the center of the base of the cone, σ=the length of the dog's path, s=the length of the projection of the dog's path on the plane (x, y) or the base of the cone, r=radius vector of this projection, a=100 feet=altitude =radius of base, m=6 feet per second, n=5 feet per second, $n/m=u$, $x^2+y^2=z^2$ is the equation of the cone. Then $u\sigma=a\theta$, where $\theta=\angle COB$, subtended by the fox's path at the center O.



$$d\sigma=(a/u)d\theta=\sqrt{(dx^2+dy^2+dz^2)}=\sqrt{(ds^2+dz^2)}$$

$$=[r^2+(dr/d\theta)^2+(dz/d\theta)^2]^{\frac{1}{2}}\,d\theta.$$

But $r^2=x^2+y^2=z^2$. $\therefore\ dz=dr$.

$\therefore d\sigma/d\theta = a/u = [r^2 + 2(dr/d\theta)^2]^{\frac{1}{2}}$, or $d\theta/dr = \left(\frac{2u^2}{a^2 - r^2u^2}\right)^{\frac{1}{2}} = A$.

Let P be the projection of the dog's path on the plane (x, y) at any time; C, the position of the fox at the same time; QC, the tangent at P; $OS = x$; $PS = y$; $OR = t$; $PC = l$; $OP = r$; $\angle POS = \varphi$; $\angle CQS = \psi$. Then $u\sigma = a\theta = a\cos^{-1}(t/a) = a\cos^{-1}[(x + PR')/a]$. But $PR' : CR' = QS : PS$. $\therefore PR' = l\cos\psi = l dx/ds$.

$\therefore u\sigma = a\theta = a\cos^{-1}[(x + l dx/ds)/a]$ or $a\cos(u\sigma/a) = x + l dx/ds$.

Also $a\cos\theta = x + l dx/ds$. But $a^2 = r^2 + l^2 - 2lr\cos OPC = r^2 + l^2 + 2lr\cos(\varphi - \psi)$, $\cos(\varphi - \psi) = dr/ds$. $\therefore a^2 = r^2 + l^2 + 2lr dr/ds$.

$\therefore l = \sqrt{[a^2 - r^2 + r^2(dr/ds)^2]} - r dr/ds$.

But $x = r\cos\varphi$, $y = r\sin\varphi$, $ds/dr = \sqrt{[1 + r^2(d\varphi/dr)^2]}$.

$$\frac{dy}{dx} = \frac{\sin\varphi\, dr/d\varphi + r\cos\varphi}{\cos\varphi\, dr/d\varphi - r\sin\varphi}, \qquad \frac{d\varphi}{dr} = \frac{x dy/dx - y}{\sqrt{(x^2 + y^2)(x + y dy/dx)}}$$

$$\text{and } \frac{ds}{dr} = \frac{\sqrt{\{(x^2 + y^2)[1 + (dy/dx)^2]\}}}{x + y dy/dx}.$$

$$\therefore l = \frac{\sqrt{\{a^2[1 + (dy/dx)^2 - (y - x dy/dx)^2]\}} - (x + y dy/dx)}{\sqrt{[1 + (dy/dx)^2]}}.$$

$$\therefore a\cos(u\sigma/a) = a\cos\theta = x + \frac{\sqrt{\{a^2[1 + (dy/dx)^2 - (y - x dx/dx)^2]\}} - (x + y dy/dx)}{1 + (dy/dx)^2}.$$

We also have $l^2 = a^2 + r^2 - 2ar\cos(\theta - \varphi)$. This establishes a relation between θ and φ; but it does not simplify the operation any. The differential equation above is too complicated for solution. If we assume that $\theta = \varphi$, and therefore, that the dog is always on a straight line between the fox and the vertex of the hill, the distance he runs can be found; for $d\varphi/dr = d\theta/dr = \frac{x dy/dx - y}{r(x + y dy/dx)} = A$.

$l = a - r$, $x = r\cos\theta = r\cos(u\sigma/a)$, $y = r\sin\theta = r\sin(u\sigma/a)$. Let $u\sigma/a = \beta$.

$$\text{Then } \frac{dy}{dx} = \frac{Arx + y}{x - Ary} = \frac{Ar\cos\beta + \sin\beta}{\cos\beta - Ar\sin\beta}. \qquad dx/ds = \frac{\cos\beta - Ar\sin\beta}{\sqrt{(1 + A^2r^2)}}.$$

Then $a\cos(u\sigma/a) = x + l dx/ds$ becomes

$$a\cos\beta = r\cos\beta + \frac{(a - r)(\cos\beta - Ar\sin\beta)}{\sqrt{(1 + A^2r^2)}}. \qquad \therefore \frac{\cos\beta}{\cos\beta - Ar\sin\beta} = \frac{1}{\sqrt{(1 + A^2r^2)}}.$$

$$\therefore \tan\beta = \frac{1 - \sqrt{(1 + A^2r^2)}}{Ar} \text{ or } \beta = \tan^{-1}\left(\frac{1 - \sqrt{(1 + A^2r^2)}}{Ar}\right). \quad \text{Now } \beta = u\sigma/a,$$

$$A=\frac{u\sqrt{2}}{\sqrt{(a^2-r^2u^2)}}. \quad \therefore \sigma=\left[\frac{a}{u}\tan^{-1}\left(\frac{\sqrt{(a^2-r^2u^2)}-\sqrt{(a^2+r^2u^2)}}{ru\sqrt{2}}\right)\right]_{r=0}^{r=a}$$

$$\therefore \sigma=\frac{a}{u}\tan^{-1}\left(\frac{\sqrt{(1-u^2)}-\sqrt{(1+u^2)}}{u\sqrt{2}}\right)=120\tan^{-1}\left(\frac{\sqrt{11}-\sqrt{61}}{5\sqrt{2}}\right)$$

$$=314\tfrac{1}{3} \text{ feet, nearly.}$$

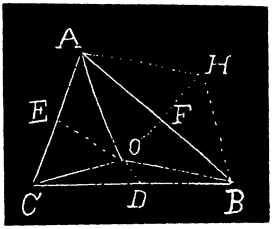
Prof. G. W. Greenwood solves the problem, interpreting it according to Prof. Zerr's assumption, but he gets a slightly different result. The Proposer furnished a partial solution for *The Educational Times*, London, obtaining the same differential equation as that obtained by Prof. Zerr, but by a different method.

MECHANICS.

150. Proposed by G. B. M. ZERR. A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

O is a point in a plane of a triangle, ABC, and D, E, F are the mid-points of the sides. Show, geometrically, that the system of forces OA, OB, OC is equivalent to the system OD, OE, OF.

Solution by G. W. GREENWOOD, B. A., Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.



Complete the parallelogram $AOBH$, of which OE is one-half the diagonal.

Since the forces OH, OB, are equivalent to OH, $\frac{1}{2}OA$ and $\frac{1}{2}OB$ will be equivalent to OF.

Similarly, $\frac{1}{2}OB$ and $\frac{1}{2}OC$ will be equivalent to OD, and $\frac{1}{2}OC$ and $\frac{1}{2}OA$ will be equivalent to OE.

Hence the system OA, OB, OC is equivalent to the system OD, OE, OF.

151. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

An elastic ball is projected along a horizontal tube, striking first the bottom, then the top, then the bottom and so on. Find the number of times the ball will strike the top.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let v=velocity just before first impact, r=diameter of tube, $v_1=\sqrt{(2gr)}$, e=coefficient of restitution. Then velocity of leaving bottom=ev, velocity of arrival at top=$ev-v_1$, velocity of arrival at bottom=$e^2v-ev_1+v_1$, velocity of second arrival at top=$e^3v-e^2v_1+ev_1-v_1$, velocity of third arrival at top=$e^5v-e^4v_1+e^3v_1-e^2v_1+ev_1-v_1$.

$\therefore$ Velocity of nth arrival at top=$e^{2n-1}v-v_1(e^{2n-2}-e^{2n-3}+e^{2n-4}-e^{2n-5}+\ldots.+e^4-e^3+e^2-e+1)$, and this velocity=$v_1$.

$\therefore e^{2n-1}v-v_1(e^{2n-2}+e^{2n-4}+\ldots.+e^4+e^2)+v_1(e^{2n-3}+e^{2n-5}+\ldots.+e^3+e)=2v_1$.